

Solutions to Maths workbook - 1 | Quadratic Equations

Level – 3

Daily Tutorial Sheet – 16

174. Here, $\log_{2x+3}(2x+3)(3x+7) = 4 - \log_{3x+7}(2x+3)^2$

or $\log_{2x+3}(2x+3) + \log_{2x+3}(3x+7) = 4 - 2\log_{3x+7}(2x+3)$

or $1 + \log_{2x+3}(3x+7) = 4 - \log_{3x+7}(2x+3)$.

Let $\log_{2x+3}(3x+7) = y$; then $\log_{3x+7}(2x+3) = \frac{1}{y}$.

$\therefore$ The equation becomes $1 + y = 4 - \frac{2}{y}$

or $y^2 = 3y - 2$ or $y^2 - 3y + 2 = 0$

or $y = 2, 1 \Rightarrow 3x+7 = (2x+3)^2, 2x+3 \Rightarrow x = \frac{-1}{4}, -2, -4$

but $x = -2, -4$ doesn't lie in domain.

Hence, $x = \frac{-1}{4}$

175. $(a + \sqrt{a^2 - 1})^{x^2 - 2x} + (a - \sqrt{a^2 - 1})^{x^2 - 2x} = 2a$

We know, $(a + \sqrt{a^2 - 1})(a - \sqrt{a^2 - 1}) = a^2 - (a^2 - 1) = 1$

$\therefore a - \sqrt{a^2 - 1} = \frac{1}{a + \sqrt{a^2 - 1}} \quad \therefore (a + \sqrt{a^2 - 1})^{x^2 - 2x} + \left(\frac{1}{a + \sqrt{a^2 - 1}}\right)^{x^2 - 2x} = 2a$

or $\left\{ \left(a + \sqrt{a^2 - 1}\right)^{x^2 - 2x} \right\}^2 - 2a \left(a + \sqrt{a^2 - 1}\right)^{x^2 - 2x} + 1 = 0$

$\therefore \left(a + \sqrt{a^2 - 1}\right)^{x^2 - 2x} = \frac{2a \pm \sqrt{4a^2 - 4}}{2} = a \pm \sqrt{a^2 - 1} = a + \sqrt{a^2 - 1}, \left(a + \sqrt{a^2 - 1}\right)^{-1}$

$\therefore x^2 - 2x = 1, -1 \quad \therefore x^2 - 2x - 1 = 0 \quad \therefore x = \frac{2 \pm \sqrt{4 + 4}}{2}$

$x = 1 \pm \sqrt{2}$ also $x^2 - 2x + 1 = 0 \Rightarrow x = 1$

$\therefore$ The solutions are $x = 1, 1 \pm \sqrt{2}$ which are independent of a ($a > 1$ ensures $\sqrt{a^2 - 1}$ is real).

176. Here $2^{\sin^2 x} + 5 \cdot 2^{1 - \sin^2 x} = 7$

Putting $2^{\sin^2 x} = y$, the equation becomes $y + 5 \cdot \frac{2}{y} = 7$

or $y^2 - 7y + 10 = 0$ or $(y - 5)(y - 2) = 0 \quad \therefore y = 5, 2$

$\therefore 2^{\sin^2 x} = 5, 2$; but $2^{\sin^2 x}$ has the maximum value 2^1 , i.e., 2.

$\therefore 2^{\sin^2 x} \neq 5$.

Hence $2^{\sin^2 x} = 2$

$$\therefore \sin^2 x = 1 \quad \text{or} \quad \sin x = \pm 1 = \sin \left(\pm \frac{\pi}{2} \right)$$

$$\therefore x = n\pi + (-1)^n \left(\pm \frac{\pi}{2} \right), n \in \mathbb{Z} \text{ (the set of integers).}$$

177. Here AM of $\{x-1, x-5\} = \text{AM of } \{x-2, x-4\} = x-3$

Put $x-3 = y$; then $x-1 = y+2, x-2 = y+1, x-4 = y-1, x-5 = y-2$.

$$\therefore \text{the equation becomes, } (y+2)^3 + (y+1)^3 + y^3 + (y-1)^3 + (y-2)^3 = 0$$

$$\text{or } \{(y+2)^3 + (y-2)^3\} + \{(y+1)^3 + (y-1)^3\} + y^3 = 0 \quad \text{or} \quad 2(y^3 + 12y) + 2(y^3 + 3y) + y^3 = 0$$

$$\text{or } 5y^3 + 30y = 0 \quad \text{or} \quad 5y(y^2 + 6) = 0$$

$$\therefore y = 0, \pm \sqrt{-6} = 0, \pm \sqrt{6}i \quad \therefore x = 3, 3 \pm \sqrt{6}i$$

178. Here $\sqrt{x+8+6\sqrt{x-1}} - \sqrt{x+3+4\sqrt{x-1}} = 1 \quad \dots (1)$

$$\begin{aligned} \therefore & (\sqrt{x+8+6\sqrt{x-1}} - \sqrt{x+3+4\sqrt{x-1}}) \times (\sqrt{x+8+6\sqrt{x-1}} + \sqrt{x+3+4\sqrt{x-1}}) \\ &= \sqrt{x+8+6\sqrt{x-1}} + \sqrt{x+3+4\sqrt{x-1}} \end{aligned}$$

$$\therefore x+8+6\sqrt{x-1} - \{x+3+4\sqrt{x-1}\} = \sqrt{x+8+6\sqrt{x-1}} + \sqrt{x+3+4\sqrt{x-1}}$$

$$\text{or } \sqrt{x+8+6\sqrt{x-1}} + \sqrt{x+3+4\sqrt{x-1}} = 5 + 2\sqrt{x-1} \quad \dots (2)$$

$$(1) + (2) \text{ gives, } 2\sqrt{x+8+6\sqrt{x-1}} = 6 + 2\sqrt{x-1}$$

$$\text{or } \sqrt{x+8+6\sqrt{x-1}} = 3 + \sqrt{x-1}$$

$$\therefore \sqrt{x+8+6\sqrt{x-1}} - \sqrt{x-1} = 3 \quad \dots (3)$$

$$\therefore (\sqrt{x+8+6\sqrt{x-1}} - \sqrt{x-1}) \times (\sqrt{x+8+6\sqrt{x-1}} + \sqrt{x-1}) = 3(\sqrt{x+8+6\sqrt{x-1}} + \sqrt{x-1})$$

$$\text{or } x+8+6\sqrt{x-1} - (x-1) = 3(\sqrt{x+8+6\sqrt{x-1}} + \sqrt{x-1})$$

$$\text{or } 9+6\sqrt{x-1} = 3(\sqrt{x+8+6\sqrt{x-1}} + \sqrt{x-1}) \quad \text{or} \quad 9+6\sqrt{x-1} = 3(\sqrt{x+8+6\sqrt{x-1}} + \sqrt{x-1})$$

$$\therefore \sqrt{x+8+6\sqrt{x-1}} + \sqrt{x-1} = 3 + 2\sqrt{x-1} \quad \dots (4)$$

$$(4) - (3) \text{ gives, } 2\sqrt{x-1} = 2\sqrt{x-1} \text{ which is an identity for all } x.$$

But the given relation can hold for $x \geq 1$.

$$\therefore \text{Solutions-set} = R \cap [1, +\infty) = [1, +\infty).$$

179. We know $pa^2 + qa + r = 0$ is an identity in x , i.e., the equation has three or more solutions, if $p = 0, q = 0$ and $r = 0$ simultaneously.

$\therefore$ The given quadratic equation will have three solutions if

$$a^2 - a - 2 = 0, a^2 - 4 = 0 \text{ and } a^2 - 3a + 2 = 0.$$

$$\text{Now, } a^2 - a - 2 = 0$$

$$\Rightarrow (a-2)(a+1) = 0$$

$$\therefore a = 2, -1$$

$$\text{Next } a^2 - 4 = 0$$

$$\Rightarrow (a+2)(a-2) = 0 \quad \therefore a = 2, -2$$

$$\text{Also, } a^2 - 3a + 2 = 0$$

$$\Rightarrow (a-2)(a-1) = 0$$

$\therefore a = 2, 1$ $\therefore$ for $a = 2$, the coefficients of x^2, x, x^0 are all zeroes.

$\therefore$ The required value of $a = 2$.

Now, rearranging into an equation in a we get $(x^2 + x + 1)a^2 - (x^2 + 3)a - 2x^2 - 4x + 2 = 0$.

It will be an identity in a if $x^2 + x + 1 = 0$, $x^2 + 3 = 0$ and $-2x^2 - 4x + 2 = 0$.

There is no real value of x satisfying the above equations in x simultaneously.

$\therefore$ The given equation is not an identity in a .